3.8: Inverse and Radical Functions

- We learned that the graph of the inverse function, if it exists, is the reflection of the function over $y = x$. This means that the x-intercept of one is the y-intercept of the other and the vertical asymptote of one is the horizontal asymptote of the other. We also learned that the domain of inverse function is the range of the original function and the range of the inverse function is the domain of the function itself.
- Only one to one functions have inverses. Quadratic functions are not one to one; we restrict the domain of a quadratic function to create a one to one function. For quadratic functions, we typically use the domain restriction $[x_{\text{vertex}}, \infty)$ to convert to a one to one function; the domain restriction to $(-\infty, x_{\text{vertex}}]$ create a one to one function as well.
- Conjugates: When simplifying radical functions you may want to use conjugates. The conjugate of $A+B$ is $A-B$ and vice versa. If a radical function contains $A \pm B$, then you may want to multiply both numerator and the denominator by the conjugate. (This is often used when finding the average rate of change with radical functions.)

Solving Radical Equations

Isolate one of the Radicals: Add or subtract terms from both sides of the equation to arrive at an equation with one radical on one side and the rest of the terms on the other side.

Square Both Sides (or raise to a power that neutralizes the radical): Now that one radical is isolated, raise both sides to power two. This way one of the radicals will be eliminated. Raising to power 2 for the other side of the equation MAY require a binomial calculation.

Eliminate the Next Radical if any: If the equation had more than one radical term, you may have to repeat the first and the second part.

Solve: When all radicals are eliminated, solve for the desired variable. A quadratic equation or other polynomial may be present at this stage.

Eliminate Extraneous Solutions: This stage of the work is really essential since, by squaring both sides of the equation, extraneous solutions may have been produced which we need to eliminate. Plug in the solutions you found in the original equation.

1. Find the domain and the range of **the inverse** of the following functions on the given domain restriction.

   (a) $f(x) = (x+3)^2 - 4$ on $[-3, \infty)$.

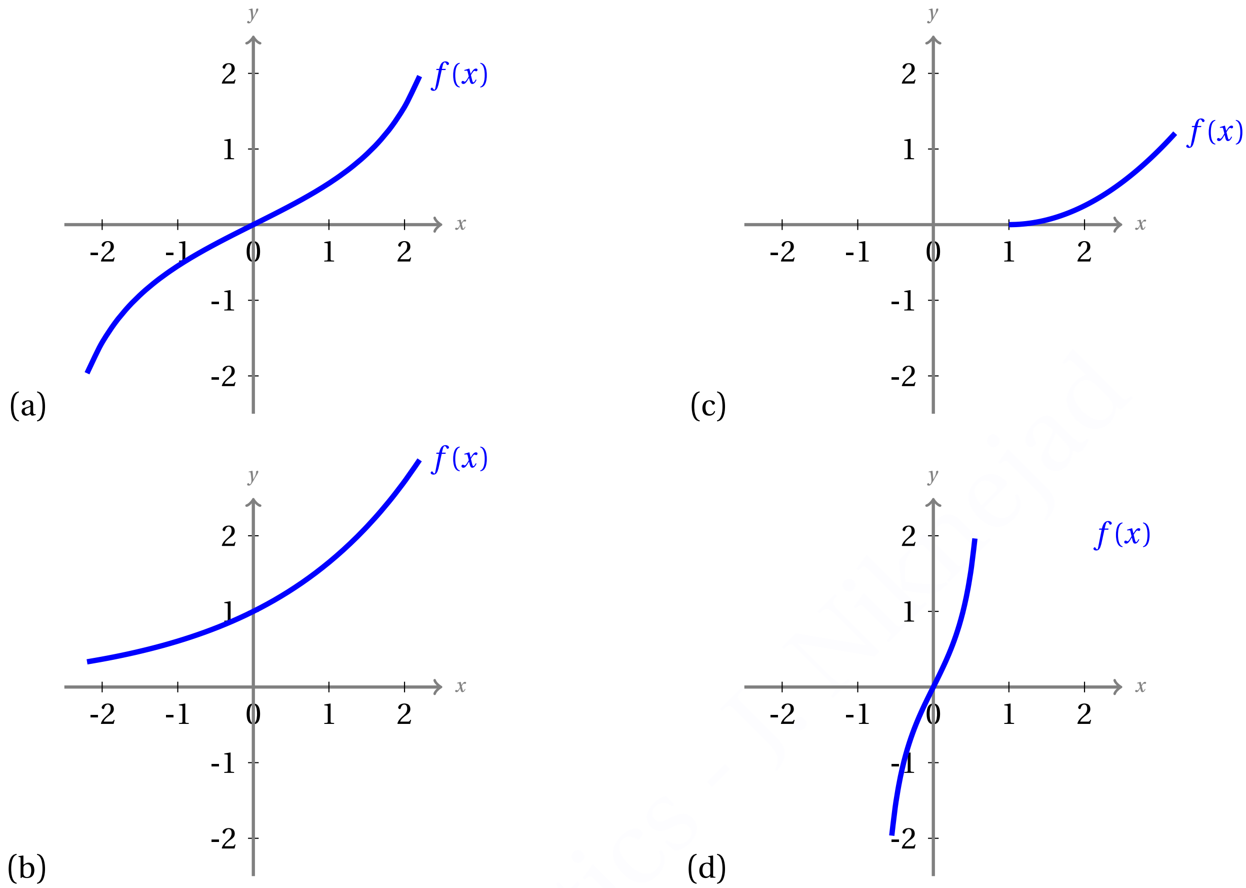
   (b) $g(x) = (x-5)^3 - 3$ on $(-\infty, \infty)$.

   (c) $h(x) = 5(x-4)^2 + 3$ on $[4, \infty)$.

2. Find the average rate of change in $f(x) = \sqrt{x}$ over the interval $[a, a+h]$; use conjugates to simplify as much as possible.

3. Find the Average rate of change in $f(x) = \sqrt{x}$ over the interval $[5, 5+h]$; use conjugates to simplify as much as possible.

4. Graph the inverse function of each function.



5. Solve $\sqrt{x-2}-6=0$ for x. Watch Gateway Video 34

6. Solve $\sqrt{2-t}=6$ for t. Watch Gateway Video 36.

7. Solve $c = 4 + \sqrt{4 - c}$ for c. Watch Gateway Video 37.

8. Solve $4x = \sqrt{56x + 312}$ for x. Watch Gateway Video 40.

9. Solve $\sqrt{6 - y} + \sqrt{5y + 6} = 6$ for y. Watch Gateway Video 43.

10. Solve $\sqrt{2x + 7} - \sqrt{2x - 9} = 2$ for x. Watch Gateway Video 42:

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